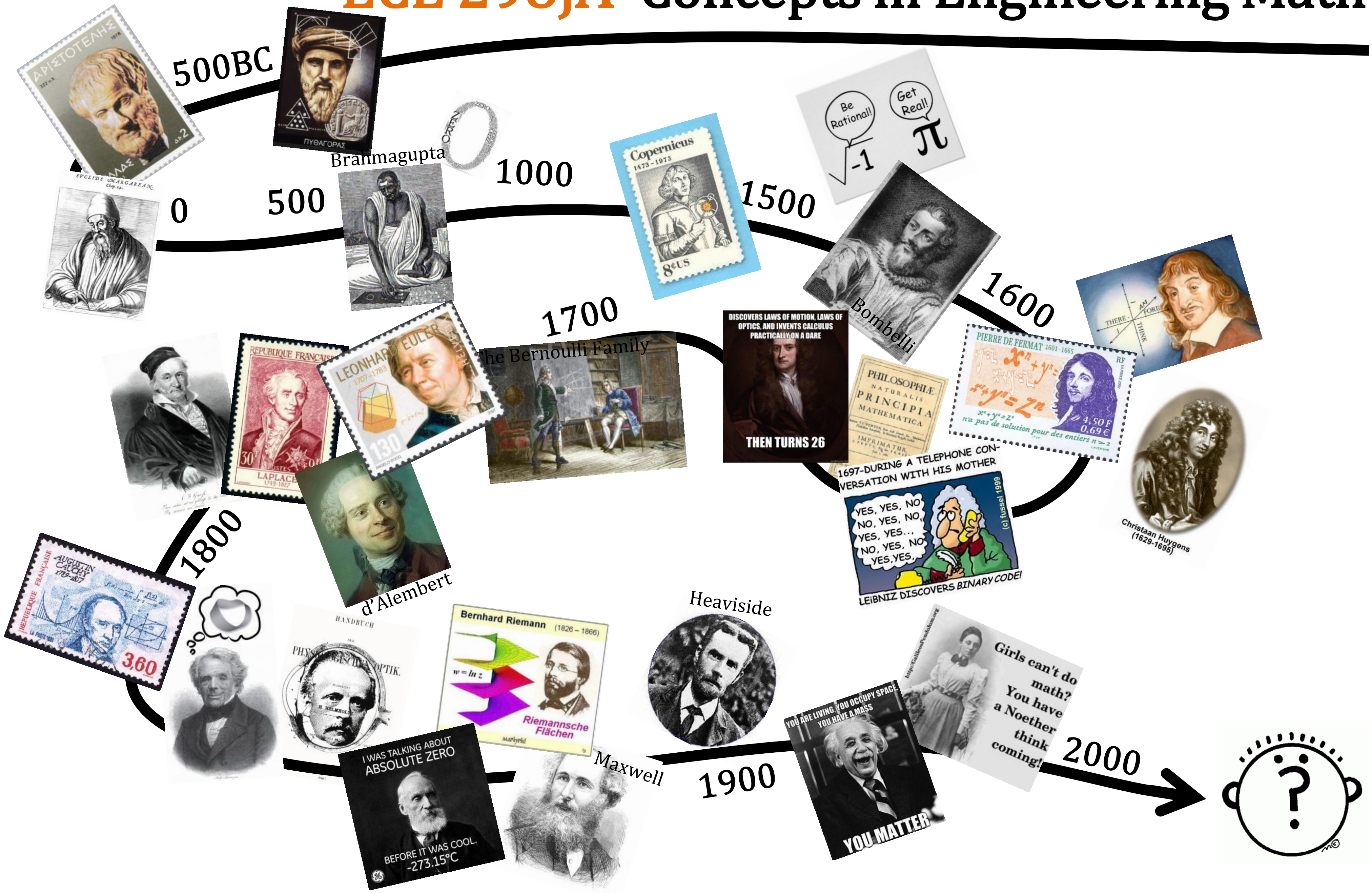
ECE 298JA Concepts in Engineering Math



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ECE-298JA: *Concepts in Engineering Math*; Fall 2015

Instructors: Jont Allen, Steve Levinson, John DAngelo (Math) and others from ECE, Math, and Physics
Course Coordinator: Jont Allen
Prerequisites: Calculus I, II. (Concurrent registration in Calc. III or Differential Equations is encouraged)
Target Audience: Sophomores & precocious freshmen from Engineering, Physics and Mathematics
Text: Stillwell, John, (2002), *Mathematics and its History;* Boas, R., (2009), *Invitation to complex analysis*

Outline: This course provides a "mathematical road map" to help students strengthen their understanding of engineering mathematics, at the conceptual level. A broad review of the development of classical mathematical theories used in contemporary engineering is presented, by emphasizing the historical discovery and development of the mathematics of linear algebra, complex analysis (e.g., frequency domain methods, impedance) and partial differential equations. This course will emphasize engineering insight and intuition building, rather than proofs. Intuitive insights into the fundamental theorems of mathematics will be presented, to help the students expand their natural creative skills. The specific mathematical contributions of Newton, Euler, Riemann, Cauchy, Gauss, Maxwell, and others, will be discussed. Problem sets will be based on engineering problems, and how they relate to classical mathematics. An extra hour of credit is given for a student project (with approval of the instructor).

ECE-298JA is presented in three parts:

I. **Number systems:** Integers, rationals, real vs. complex numbers, vectors, matrices.

II. **Algebraic equations:** Topics will include time and frequency domains (e.g., Laplace transforms), complex impedance (e.g., for a capacitor $Z(s) = 1/sC$ is a function of the complex variable $s = \sigma + j\omega$), how electrical mechanical and thermal networks are described by matrices, eigenvalues, impedance-based integral equations.

III. **Differential equations:** Ordinary differential equations for LRC circuits, Kirchhoffs laws, etc.; Partial differential equations, such as the Laplace, diffusion, wave, and Maxwell's Equations.

Course outline by topic:[1]

W	L	c.	Description
			Part I. Number systems
1	1	(50)	The discovery of Number systems
		(3)	Introduction: Integers, rationals, real vs. complex numbers, vectors, matrices.
		7	First use of zero as a number (Brahmagupta defines rules)
	2	19	Sets (Cantor); Why closing a set important? The existence of Limits.
		20	Set theory in Fourier-like Transform
		12	The first use of ∞ (Bhaskara's interpretation); The extended circle (regularization at ∞) (p. 280)
	3	3	*Recursion*: Why is the convergence of a series important?
		16	Complex numbers (Bombelli, 1575, p. 259); *Radius of convergence* (ROC)
2	4	(5)	Aristotle, Pythagoras and the *beauty of integers*
		3,9	*Eigenmodes*: Mathematics in Music and acoustics: Strings, Chinese Bells, chimes
	5	(3)	Euclid: Ruler and Compass constructions: Conic sections
	6	17	Analytic Geometry: Geometry (Euclid) vs. "algebraic roots" (Descartes, Newton)
3	7	(3)	Greek number theory: Euclid
	8	(5)	Pythagorean triplets (p. 43)
	9	(5)	Why are integers so important to the Greeks? (Eudoxus, Archimedes) (p. 57)
4	10	(3)	*Continued Fraction algorithm* (Euclid & Gauss, p. 47, GED, p. 41, 66)
	11		Complex Pell's Eq. (p. 72)
	12	8-11	Ch. 6: Polynomials (p. 82) and the first "algebra" (al-jabr)
5	13		Exam I: Number Systems

[1]W: Week; L: Lecture; c.: Century (BCE), CE; Page numbers are for Stillwell 2^d edition.

W	L	c.	Description
			Part II. Algebraic Equations
			Composition of polynomials of various degrees
	14	(10)	Chinese discover Gaussian elimination (*Jiuzhang suanshu*) (p. 84)
		7	Solution of the quadratic (Brahmagupta, 628)
		16	Solution of the cubic (c1545) (p. 91) (Tartaglia et al..., 1535)
			Bombelli first uses complex numbers, 1572 (p. 258)
	15	17	First Analytic Geometry (Fermat 1629; Descartes 1637) (p. 113)
			Computing and interpreting the roots of the characteristic polynomial (CP)
6	16	18	Root classification for polynomials of Degree $d = 1, 2, 3, 4$ (p. 96);
		17	Newton (1667) labels complex cubic roots as "impossible" (p. 112)
			Newton's "irrational" power series
		19	Quintic ($d = 5$) cannot be solved (Abel, 1826);
			3D representations of 2D systems; Perspective (3D) drawing.
	17	17	Möbius's Homogeneous Coordinates (1827) (Genus, cross-ratio) (p. 134)
	18	19	Introduction to the *Riemann sphere* (1851) (the *extended plane*) (p. 281, 291)
			Understanding ∞ by closing the complex plane
		18	Riemann's and Cauchy's role in the acceptance of Complex numbers:
			First calculus in the complex z plane: $dF(z)/dz$, $\int F(z)dz$
7	19	19	Möbius transformation; Ratios of polynomials (Poles & zeros)
			ABCD Matrix composition; Commuting vs. Noncommuting operators
	20		Why do we use *complex vector functions* of $s = \sigma + j\omega$ in Engineering and Physics?
			A.E. Kennelly introduces complex impedance, 1893;
	21		*Fourier Transforms* (Hilbert space) for signals vs. *Laplace transforms* for systems;
			The Role of Causality; Why $2\tilde{u}(t) \equiv 1 + \text{sgn}(t) \leftrightarrow 2\pi\delta(\omega) + 1/j\omega$ is not causal
			Utility of functions of a complex variable
8	22	20	The 6 postulates of System (aka, Network) Theory: The role of the causality postulate
			Fundamental limits of the Fourier vs. Laplace Transform: $\tilde{u}(t)$ vs. $u(t)$
	23	19	The Cauchy's *Residue theorem* (p. 299); Green's Thm in the plane
			The Inverse Laplace transform; poles and zeros; Residue expansions; $\frac{u(t)}{\sqrt{\pi t}} \leftrightarrow \frac{1}{\sqrt{s}}$
			The relation between the Laplace transform and a convergent power series
			Complex-analytic series representations: (1 vs. 2 sided); ROC of $1/(1-x^2)$
	24		Exam II: Systems of equations

W	L	c.	Description
			Part III. Differential Equations
			Calculus and power series
9	25	17	Calculus (p. 146); Newton & Leibniz
			Series (zeros), partial fractions (poles), products (4 formats)
	26		Implicit differentiation (p 151), rational fractions (p. 154),
			1) Series; 2) residue; 3) pole-zero; 4) continued fraction
	27		Inverse of analytic functions; Fundamental Thms of Mathematics (Ch 9):
10	28	17	Infinite power Series and analytic function theory (p 171) as an extension of the polynomial;
	29		ROC of complex vs. real poles; Inversion of a power series; Introduction to *analytic continuation*
	30		The amazing Bernoulli family; Fluid mechanics; airplane wings; natural logarithms
		20	Noether's theorem: time invariance and conservation of energy
11	31		Generating functions and the z transform: The notion of *delay*.
			Continued fractions (long division) (p. 183): $\left(1+\sqrt{5}\right)/2 = 1 + 1/1 + 1/1 + /1 \ldots$
	32	18	Analytic functions: Euler's vs Riemann's Zeta Function (i.e., poles at the primes);
			$\tan^{-1}(z) = \frac{1}{2i} \ln \frac{i-z}{i+z}$ as the "inverse" of Euler's formula 1748 (p. 295)
	33	18	Euler: The beginning of modern mathematics.
12	34		The multiplication of polynomials as convolution
			Generalizations of Pascal's triangle (the 3-headed coin)
	35	17	Fermat's last thm (Guest Lecture, for fun)
	36	19	Complex analytic functions of Genus 1 (p. 343)
13	37		Multi-valued "functions" (and their many inverses!); branch cuts
			A real-world example where the branch-cut placement is critical
			Partial differential equations of Physics
	38	17	The wave equation: Newton, d'Alembert, Descartes
		19	Transmission matrices and Möbius composition
			Möbius composition as non-commuting operators
	39	20	Wave vs. Diffusion equations (*Einstein causality*)
			J.C. Maxwell unifies Electricity and Magnetism (1861)
			O. Heaviside's vector form of MEs (1884):
			Brune's Impedance and the *quasi-static approximation* ($a \ll \lambda$)
			Wave impedances that are *not* quasi-static
			Force/velocity (like voltage/current) as an impedance $Z(s)$
			Irrational impedances: (e.g., diffusion and the skin effect $\sqrt{s}$)
14	40	20	The *quasi-static approximation* ($a \ll \lambda$) and Quantum Mechanics
			Normal modes vs. eigen-states, delay and quasi-statics
			Impedance boundary conditions (integral equations)
	41		Recap of *The Fundamental Thms of Mathematics* & their applications
	42		Review or guest lecture
			Final Exam