Loudness and the JND

An introduction to loudness Psychophysics

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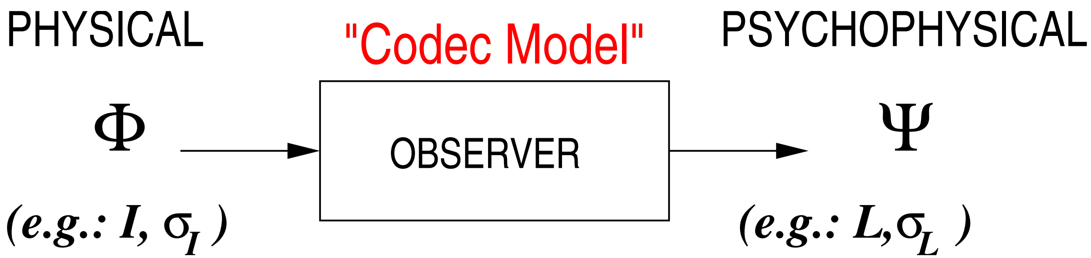
ECE-437

The intensity JND is internal uncertainty

- Perception is stochastic: Each time you hear (see) the same short tone (light) pulse, you hear (see) it with a different loudness (brightness)
- The intensity JND$_I$ ($\Delta\mathcal{I}$) is a measure of this internal perceptual fluctuation (noise) given by $\sigma_{\mathcal{L}}$



  - The *Loudness JND* $\Delta\mathcal{L} \propto \sigma_{\mathcal{L}}(\mathcal{L})$
  - The loudness JND is proportional to the internal "loudness noise"

Weber's Law (1846)

- In 1846 Weber showed experimentally that $\Delta I \propto I$
  - I is the physical intensity, and ΔI is called the *JND*
  - Weber used weights of varying relative mass
- Def: $\Delta I/I$ is called the Weber Fraction
- Def: Weber's Law says the Weber fraction is constant
- Weber's law sometimes holds:
  - Wide band noise Intensity discrimination (Miller, 1947)
  - The Weber fraction is not constant for pure tones (Riesz, 1928).
- A floating point converter obeys Weber's Law

Pure-tone intensity discrimination

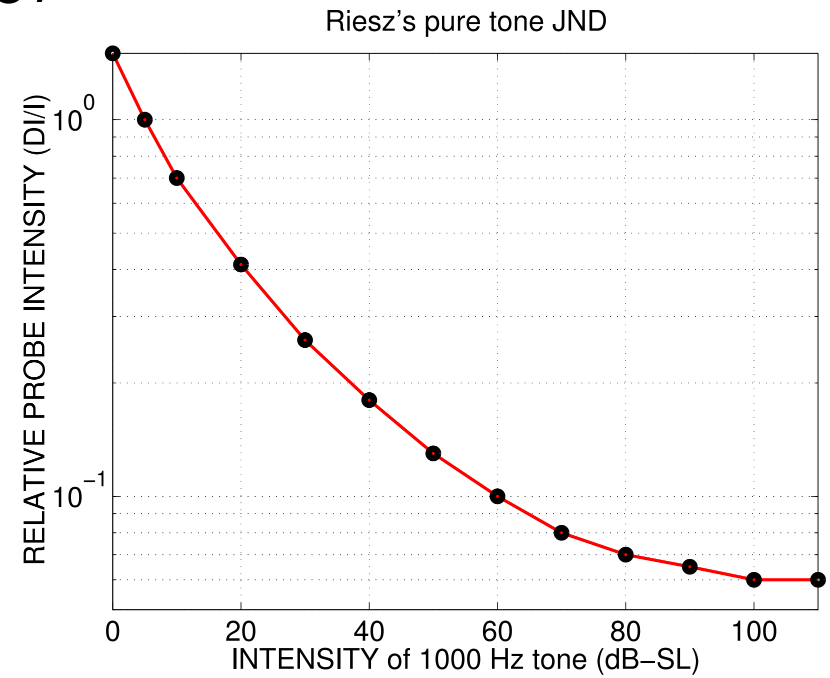
- Weber's "law" says that $\Delta I \propto I$
  - Weber's Law holds for floating point conversion
  - For fixed point, $\sigma_I = \Delta I$ is a constant
  - Is the ear a fix or floating point converter?
- **1928** Riesz establishes the near-miss to Weber's law for tones

Weber's Law (1846)

- **PROBLEM:** Weber formulated his problem in the physical domain, but the noise is internal

Near-miss to Weber's Law (1846)

- Riesz used two beating tones 3 Hz apart for this measurement (i.e., 1000 Hz masker and a low-level 1003 Hz probe)



- The near-miss to Weber's Law results from the fact that the internal noise $\sigma_{\mathcal{L}} \propto \Delta\mathcal{L}(\mathcal{L})$ is not independent of $\mathcal{L}$.
- In fact noise is Poisson-like: [Allen and Neely 1997]

$$\Delta\mathcal{L}(\mathcal{L}) \approx \sqrt{\mathcal{L}}$$

Fechner's Hypothesis (1860)

- Fechner 1860 is called the father of psychophysics.
- *Fechner's hypothesis* (or postulate) was that the loudness JND $\Delta L(I)$ is constant:

$$\Delta L(\not{I}, \not{L})$$

- Fechner assumed "that the total change in loudness between two intensities I_1 and I_2 may be found by counting the number of JNDs."
- From *Fechner's hypothesis* and the "counting formula:"

$$N_{\text{JND}} \equiv \int_{L_1}^{L_2} \frac{dL}{\Delta L} = (L_2 - L_1)/\Delta L$$

Fechner's JND theory

- Fechner's idea was that the loudness $\mathcal{L}(I)$ is proportional to the number of JND steps N_{JND}, which is given by:

$$N_{\text{JND}} \equiv \int \frac{d\mathcal{L}}{\Delta\mathcal{L}(\mathcal{L})} = \int \frac{dI}{\Delta\mathcal{I}(I)}$$

  - He assumed that $\Delta\mathcal{I} \propto I$, i.e. Weber's Law
  - He assumed that the internal noise $\Delta\mathcal{L} = \sigma_{\mathcal{L}}$ is constant

- These two assumptions give Fechner's "Law":

$$\mathcal{L}(I) \propto \log(I)$$

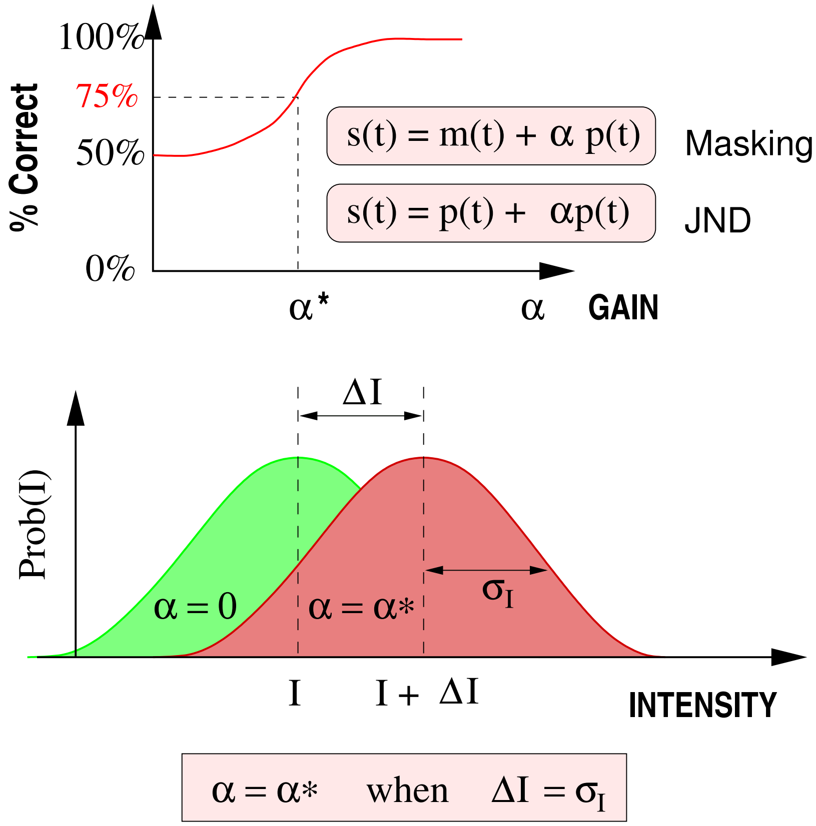
Fechner's JND theory

- Counting JNDs is a great conceptual start :◦)
- Both assumptions
  - Weber's Law
  - Fechner's Hypothesis

  are wrong :◦(
- Fechner's "Law" is wrong

Theory of Signal Detection

- L. L. Thurstone 1927 and later David Green 1965: Formally define the intensity JND as "the relative signal level for detection 75% of the time"
- $\Delta I \propto \sigma_I$:



CHRONOLOGICAL DEVELOPMENT

YEAR	CONCEPT	REFERENCE
1846	**JND**	Weber
1860	**Counting JNDs**	Fechner
1927	**Decision theory model**	Thurstone
1928	**Near-miss to Weber's law**	Riesz
1933	**Masking and loudness**	Fletcher and Munson
1947	**Wide–band JND ($J = 0.1$)**	G. A. Miller
1966	**Signal detection theory**	Green and Swets
1997	**Loudness and the JND**	Allen and Neely

Loudness Additivity

- Fletcher and Munson 1933 showed that loudness adds
  - Adjust I_2 so that: $\mathcal{L}(I_1, f_1) = \mathcal{L}(I_2, f_2)$
  - Two equally loud tones, played together are twice as loud: $\mathcal{L}(I_1, I_2, f_1, f_2) = 2\mathcal{L}(I_1, f_1)$
- Find gain $\alpha(I)$ such that

$$\mathcal{L}(\alpha I_1, f_1) = 2\mathcal{L}(I_1, f_1)$$

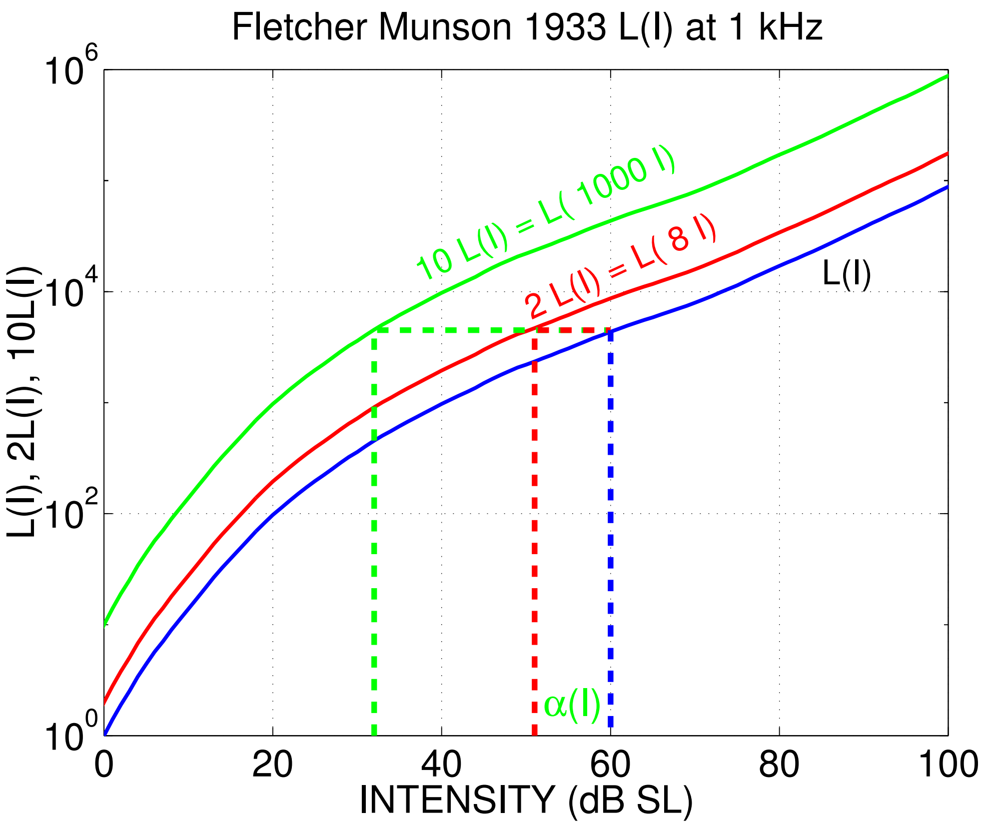
- Results: α is about 9 dB (actually it depends on intensity)

Loudness additivity

- Fletcher and Munson's **1933** loudness growth data based on loudness additivity is now called:

  Stevens' Law: $\mathcal{L}(I) = I^{\nu}$, with $\nu \approx 1/3$

- Loudness vs. intensity for 1, 2, and 10 equally loud components:



BASIC MODEL OF OBSERVER

- Transformation from $\Delta\mathcal{I}(I)$ to $\Delta\mathcal{L}(\mathcal{L})$