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Method For the Determinination of the Statistical Values of Observations Concerning Quantities Subject to Irregular Fluctuations

A. Einstein

Let us suppose that the quantity $y = F(t)$ (for example, the number of sun spots) is determined empirically as a function of time for a very large interval T. How can we characterize the statistical properties of y?

One response to this question, suggested by the theory of radiation, is the following.

Suppose that y is expanded in a Fourier series:

$$y = F(t) = \sum A_n \cos \pi n \frac{t}{T}$$

The successive coefficients A_n of the expansion will be very different from each other, in terms of magnitude and sign, and will follow each other in an irregular manner. However, if the mean value $\overline{A^2_n}$ of A^2_n is formed for a large interval Δn of the indices n, but still sufficiently small that $\pi\Delta n/T$ is also very small, this mean value will be a continuous function of n.

We will call I the *intensity* of y corresponding to n. The intensity thus defined will have a period $\theta = T/n$, and we will designate it $I(\theta)$; the problem consists of determining this function.

A simple calculation gives us:

$$I(\theta) = \overline{A^2_n} = \frac{2}{T^2}\int_0^T\int_0^T F(t)F(n) \cos \pi n \frac{t-n}{T}\, dn\, dt$$

From this it follows that the unknown function I can be determined, to within a numerical factor, by the following rule.

We select a time interval Δ and form the mean value:

$$\mathfrak{M}(\Delta) = \overline{F(t)F(t+\Delta)} \qquad (1)$$

which, for the given function y, is a characteristic function of Δ. For large values of Δ, $\mathfrak{M}(\Delta)$ will tend towards a limit which can be made zero by an appropriate translation of the axis of the abcissas (t and Δ axes). We thus have:

$$I(\theta) = \int_0^T \mathfrak{M}(\Delta) \cos \pi \frac{\Delta}{\theta}\, d\Delta \qquad (2)$$

We already know the mechanical means for carrying out the integration indicated in Eq. (2). My friend, Mr. P. Habicht, also showed me that the mean values of Eq. (1) can be determined easily using a simple mechanical integrator. The practical execution of the method thus seems to offer no particular difficulty.

Furthermore, we will note that an integrator which permits us to obtain mean values as in Eq. (1) can also be used to answer the following question: Is there a cause-and-effect relationship between two quantities F_1 and F_2 which are both determined empirically as a function of time? If we form

$$\mathfrak{M}(\Delta) = \overline{F_1(t)F_2(t+\Delta)} \qquad (3)$$

as a function of Δ, we obtain a horizontal straight line for $\mathfrak{M}(\Delta)$ if there is no cause-and-effect relationship. If such a relationship exists, without any appreciable delay, we obtain a curve which has an extremum at $\Delta = 0$. If there is an appreciable delay, the curve has an extremum at some other value of Δ.

Mr. Weiss remarked how the method developed by A. Einstein will contribute greatly to meteorological research, where a large fund of data has, up to the present time, remained largely unexploited.

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