ECE-208–*Concepts in Engineering Math*

Instructor & **Course Coordinator:** Jont Allen
Prerequisites: Differential Equations
Target Audience: Second and third year Engineering undergraduates
Text: *An invitation to mathematical physics and its history*, Jont Allen, Springer 2020
https://link-springer-com.proxy2.library.illinois.edu/book/10.1007/978-3-030-53759-3
Potential instructors: Mark Hasegawa-Johnson; Zhi-Pei Liang, Minh Do

Outline: This course covers the formulation and solution of problems using complex linear algebra and the scalar wave equation, allowing for complex eigen-values and wave causality. The theory of the Laplace transform and its inverse are properly developed, based on the *Fundamental theorem of complex calculus* (FTCC), the *Cauchy-Riemann conditions* (CR) and the *Cauchy residue theorem*.

I. Introduction and History: Roots of polynomials, the companion matrix and Eigen-analysis methods with a comparison of the properties of Fourier & Laplace transforms.

II. FTC & FTCC; Complex analytic functions and their solution based on the Laplace transform and it's inverse.

Assignments Problems drawn from (See text): **NS2**, **AE1**, **AE2**, **AE3**, **DE1**, **DE2**, **DE3**

Assignment solutions: The solutions to the HWs are posted before the assignment is due. Students are expected to attempt the solutions on their own, but before the clock runs out, they are invited to review the solution.

Exams: The exams are constructed from the HWs.

Syllabus: https://auditorymodels.org/index.php?n=Courses.ECE298-ComplexLinearAlg-F21

Part I. Introduction to 2x2 matrices (8 Lectures)			
Wk	M	W	F
1	Introduction	Importance of polynomials	**NS-1**
2	Roots of polynomials	Newton's method (and its issues)	
3	The companion matrix	Properties of the eigen matrix	**AE-1**
4	Pell's Eq	Fibonacci Series	
5	Eigen-Analysis	Eigen-Analysis	**AE-2**
6	Brune (physical) Impedance	Key properties of impedance/admittance	**AE-3**
7	Fourier transforms ($\mathcal{FT}$)	Laplace transforms ($\mathcal{LT}$)	Exam

Part II. Complex algebra (12 Lecs)			
Wk	M	W	F
8	Laplace transform ($\mathcal{LT}$) & Definitions	Examples	
9	Complex analytic functions and Taylor series	FTC vs. FTCC & Integration in $\mathbb{C}$	**DE-1**
10	Cauchy-Riemann conditions	convolution & causality	
11	Multi-valued functions & Branch cuts	domain coloring (zviz.m) & Branch cuts	**DE-2**
12	Riemann's extended plane	Schwarz inequality & the Triangle inequality	
13	3 Cauchy integral Thms CT-1,-2,-3	Inverse $\mathcal{LT}^{-1}$ & the role of the *R*esidue	
14	The wave equation	Train-mission-line problem (p. 262)	**DE-3**
15	Review for Final	Introduction to Maxwell's Eqs. (optional)	
Final			

Homework:	10%
Midterm:	35%
Final:	55%

Objectives and Justification

This course has been taught eight times as ECE-298CLA, and three times as a full semester course as ECE298JA. Click on `>Teaching` in the left-sidebar of the class website `https://auditorymodels.org`. The course was transformed into a half-semester 2 credit hour course in 2014. An option is provided to sign-up for an hour of student research ECE-297, for those who wish extra credit.

The course complements ECE-210 and ECE-310 with a more detailed treatment of the Laplace transform ($\mathcal{LT}$). It may be taken at the same time or after ECE-210.

The course introduces key mathematical topics, such as complex analytic functions, the Cauchy residue theorem and the Cauchy-Riemann conditions. The inverse Laplace transform $\mathcal{LT}^{-1}$ critically depends on the *Cauchy residue theorem* (CR-1, CR-2), thus forcing causality. Once the laws of electrical circuit and Newton's laws are transformed into the complex frequency $s = \sigma + \jmath\omega$ (the $\mathcal{LT}$ domain), calculus becomes algebra.

Physical insight (not math) is the key to understanding such problems. For example, the real part of the eigenvalue σ represents the damping while the imaginary part $\jmath\omega$ represents the resonant frequency.

All the concepts of linear-algebra become obvious once the *companion matrix* is introduced.

Overview

MATH courses that have related material cover two topic areas:

Linear algebra

These include the following courses in MATH
225: *Introduction to Matrix theory*
227: *Linear Algebra for Data Science*
285, 286: *Intro Differential Equations, Eq. Plus*
415: *Applied Linear Algebra*
442: *Intro Partial Diff Eqs*
487: *Advanced Engineering Mathematics*
There are no 200 level course on complex analysis in the MATH curriculum. These math courses do not discuss the Laplace transform. Fourier transform (not Laplace transform) methods are mentioned in 285, 286 and 442. MATH-487 is cross-listed with ECE-493, which is heavily overlapped with ECE-208. The important question of complex roots of polynomials (e.g., The relevant *Fundamental theorem of algebra* is not mentioned), all of which are presented in MATH-487 and the proposed ECE-208, with examples in terms of complex impedance $Z(s) = R(\sigma, \omega) + \jmath X(\sigma, \omega)$ as a function of the complex Laplace frequency $s = \sigma + \omega\jmath$.

Complex analysis

446: *Applied Complex Variables*
Integration "by residues" and potential fields are presented but not complex roots of polynomials. For a "systematic development" students are directed to MATH 448
448: *Complex Variables*
This course specifically deals with Cauchy's theorem, the residue theorem and the Fundamental theorem of algebra
487: *Advanced Engineering Mathematics* (Cross-listed with ECE-493)

Summary

The fundamental topic of complex linear algebra require taking two non-overlapping 400-level MATH courses. Also to understand the theory of the inverse Laplace transform, one must take the equivalent of MATH 448. This course would normally be taken by 4th year students, but typically *not* by second or even third year students. Thus ECE-208 is need to properly prepare the engineering student.